COORDINATION *and* CULTURE

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Abstract

We analyze social coordination when agents are bound by cultural commitments – agents only consider a subset of the action set, ruling out other actions as being culturally impermissible (i.e. taboo). When individuals choose from different action sets, do unified social conventions emerge? If agents can choose their action sets, would they choose more or less restrictive commitments? Using techniques from stochastic evolutionary game theory (Young 1993, 1998; Ellison 2000), we uncover a completely novel long-run possibility, akin to a clash of cultures, in which coordination between cultures permanently breaks down, even when coordination is Pareto-efficient. There exist distributions of individual preferences for which recurrent miscoordination is stochastically stable. Therefore, cultural commitments can explain why social diversity persists despite large benefits to homogenization of attitudes and behavior. We also show that more restrictive (i.e. intolerant) cultures can survive evolutionary selection when agents choose their culture. In a society which begins with three cultural communities, at most two cultures survive in the long run: one restrictive and one permissive.

23 November 2008

JEL Classification: C72, C73, Z1

Keywords: Coordination Games, Cultural Evolution, Social Dynamics

*This research was supported by a General Sir John Monash Award, sponsored by the Commonwealth Bank Foundation. The author is heavily indebted to Peyton Young for his guidance and support and to David Myatt. The paper has also benefitted from comments by Rob Boyd, Eric Engler, Herb Gintis, Courtney Monk, Ryan Muldoon, Tom Norman and Chris Wallace. The usual caveat applies.

To compare is to destroy. Merely making explicit the possibility of certain trade-offs weakens, corrupts, and degrades one's moral standing. Fiske & Tetlock (1997)

1 Introduction

A fundamental feature of economic theory is that individuals make choices by comparing the consequences of alternative actions. However, empirical research indicates that there are certain comparisons which people tend to reject. Tetlock et al. (2000) find that experimental subjects express moral outrage at even contemplating certain taboo transactions, including buying and selling of human body parts for medical transplant operations, surrogate motherhood contracts, adoption rights for orphans, votes in elections for political office, the right to become a U.S. citizen and sexual favors (i.e. prostitution). People feel that contemplating taboo trade-offs undermines their image as moral beings both to themselves and other members of society (Fiske & Tetlock 1997). We model an agent's repugnance to comparing the consequences of certain alternatives as a restriction on their choice set. Let X be the global set of actions. An individual k only considers taking actions in $X_k \subseteq X$. Roth (2007) discusses how widespread repugnance becomes enshrined in law and acts as a constraint on markets (e.g. market for organs). In this paper, we examine the implications for social coordination of people having different sets of taboos or feasible action sets. When individuals choose from different action sets, do unified social conventions emerge in which agents take actions that are mutually admissible? If agents can choose their action sets, would they choose more or less restrictive action sets?

We employ techniques from stochastic evolutionary game theory (Foster & Young 1990, Young 1993, Ellison 2000) to show that common *conventions* can emerge in intercultural interactions, even when agents choose from different action sets. This result holds in the presence of far greater heterogeneity in preferences than analyzed in previous work. However, social coordination can *permanently* break down, even when coordination is Pareto-efficient. There exist distributions of preferences for which recurrent miscoordination is stochastically stable. This phenomenon is clearly different to the selection of a Pareto-inefficient coordination equilibrium when agents use the same action set (Kandori et al. 1993, Young 1993). Here, play might not settle into any coordination equilibrium, even though every coordination equilibrium Pareto dominates miscoordination.

Tetlock et al. (2000) find that moral outrage at taboo transactions varies across subjects in a systematic way, depending on agents' political attitudes. More generally, we can think of agents belonging to K disjoint cultural communities.[1] An adherent to culture c_k is committed to taking actions in $X_k \subseteq X$. As such, we develop a model of coordination between cultures, in which agents are randomly matched to play a coor-

[1]With regard to issues of commitment, Sen (1977) explains that "in a broader sense these are matters of culture, of which morality is one part" [p. 334]. He adds that "[g]roups intermediate between oneself and all, such as class and community, provide the focus of many actions involving commitment" [p. 344].

dination game, and characterize the conditions under which a 'clash of cultures' can emerge (Huntington 1993). Cultural commitments explain why social diversity exists despite large benefits to homogenization of attitudes and behavior. In addition, we show that more restrictive (i.e. intolerant) cultures can survive evolutionary selection when agents can choose their culture.

Cultural explanations appear in the work of Adam Smith, Thomas Malthus, Karl Marx, Max Weber and Joseph Schumpeter. Yet modern economic theory has had little to say about culture. Economic historians have grappled with the way in which culture shapes and is shaped by economic change (Lal 1998, Landes 1998, North 2005, Jones 2006). Both authors point to the absence of a firm theoretical foundation. Jones regrets that "[f]ew available generalizations are guides to which aspects of culture are more likely to alter and which are likely to persist, what causes change and what accounts for stasis, which individuals will readily adapt and which will not" [p. 48]. Our model contributes to an account of cultural change.

The simple notion of culture we introduce involves an adherent to culture c_k being *committed* to taking actions in $X_k \subseteq X$. We suggest that this commitment arises from the internalization of culturally defined standards of behavior through the process of *socialization*. Another formulation which is completely consistent with our analysis is where culture k shapes preferences such that all $x \notin X_k$ are strictly dominated choices. With this in mind, we adopt the first interpretation of culture. According to Boyd & Richerson (2005, p. 3):

> *Culture is information that people acquire from others by teaching, imitation and other forms of social learning.* On a scale unknown in any other species, people acquire skills, beliefs, and values from the people around them, and these strongly affect behavior. People living in human populations are heirs to a pool of socially transmitted information that affects how they make a living, how they communicate, and what they think is right and wrong. [*emphasis in original*]

During socialization, standards of "right" and 'wrong" are internalized by the individual (e.g. Child 1943, Durkheim 1953, Merton 1957).[2] According to Coleman (1990), the internalization of behavioral norms means that individuals come to have an "internal sanctioning system" which provides punishment when they violate standards of acceptable behavior [p. 293]. In other words, individuals incur a psychological cost from taking actions that are deemed to be wrong (e.g. Frank 1988, Elster 1989, Akerlof & Kranton 2000).[3] Frank (1988) argues that emotions such as shame, guilt, and anger enforce moral commitments, making credible the carrying out of threats and promises even when it is not in a person's material interest to do so. In this study, we assume that emotional cues are determined by an individual's culture. So culture animated by the emotions plays the role of a commitment device, constraining individual be-

[2]Henrich et al. (2004) present evidence of the internalization of culturally defined standards of behavior from their experiments in 15 small-scale societies spread over five continents.

[3]We do not explicitly model this psychological loss here. Of course, there may be other forces which preserve these rules of conduct, such as the imposition of sanctions by the community, which we do not model here. Nevertheless, as Rao & Walton (2004) claim regarding the ubiquitous incest taboo (e.g. Freud 1950), "[m]ost people would not consider breaking it, not just because of fear of social sanctions, but simply because the taboo is so deeply ingrained within their psyches" [p. 15].

havior and generating what Sen (1977) aptly calls "commitments" and what Harsanyi (1982) calls "moral values". Clearly, individuals are not always compliant, and deviant behavior is observed in real social settings. Nevertheless, individuals that reject social controls and experience no guilt or remorse when transgressing socially acceptable standards of behavior constitute only 3-4 per cent of the male population and less than 1 per cent of the female population in the United States (Mealey 1995). These individuals are known as sociopaths (or psychopaths) in the literature.

Our model could be applied to a variety of contexts, including technology adoption and language choice. For example, the extreme devotion exhibited by Macintosh users to Apple products (see Belk & Tumbat 2005) can be modeled as a cultural commitment. This could explain why Apple Macintosh systems are still used, despite large network externalities favoring the adoption of the more popular MS Windows system. Cultural commitments may also explain the diversity of languages spoken in countries such as Spain, Switzerland and India, as well as the persistence of local dialects in the face of regional and global integration (Fishman 1991). To illustrate more clearly the type of cultural commitments we have in mind, however, let us focus for the moment on the example of cultural assimilation. Until the 1960s, social scientists were predicting that immigrants to countries such as the United States would rapidly assimilate into the culture of their host country. On the contrary, the cultural background of second-generation immigrants tends to predict the norms of behavior to which they subsequently adhere, including female labor supply, fertility decisions and living arrangements (Fernandez & Fogli 2005, Giuliano 2007). Around the world, ethnic minorities, such as the Jewish diaspora, Irish Catalans and Basques, persistently adhere to distinct norms of behavior (see Bisin & Verdier 2000). Veiling among Muslim immigrants in Europe and America is one of the most potent symbols of cultural difference. Bisin et al. (2007) find that a Muslim born in the UK and having spent more than 50 years there is approximately as likely to exhibit a strong religious identity as a non-Muslim who has just arrived in the country. Our theory illustrates how the strict Islamic moral code which generates cultural commitments and divergent preferences could lead to persistent deviations from majority norms despite large individuals benefits to assimilation.

Consider the following model. Individuals are drawn from two disjoint communities to engage in social interactions. Prior to interacting they both choose a standard of dress $x \in X$, where lower values of x indicate greater modesty (e.g. veiling). However, agents from community 1 reject any standard of dress which is too immodest $x > \overline{x}$. Agents from community 2 reject any standard of dress which is too modest $x < \underline{x}$. All agents benefit from adopting the same standard of dress in social interactions. Coordination between members of the two communities is possible as long as $\underline{x} < \overline{x}$. However, we shall show that even if there exists an $x \in [\underline{x}, \overline{x}] \cap X$ and coordination is Pareto-efficient, an equilibrium can emerge in which all agents in community 1 adopt a modest standard of dress $x < \underline{x}$ and agents in community 2 choose an immodest standard of dress $x > \overline{x}$. Thus social coordination breaks down, and cultural diversity persists despite large benefits from homogenizing social behavior. This can occur, if absent any benefit from coordination, the most preferred choice of dress is less than $\underline{x}$ for agents in community 1 and greater than $\overline{x}$ for agents in community

2. The intuition is that when the two communities have had a history of miscoordination, no one expects coordination in the future. Thus all agents choose their most preferred choice, disregarding any benefits from coordination. In this way, miscoordination is perpetuated. Therefore, recurrent miscoordination requires both different sets of cultural commitments (i.e. action sets) as well as diverging preferences between cultural communities. If preferences are sufficiently divergent, we show that recurrent miscoordination is stochastically stable.

As Sen (2006) has argued, inheritance is not destiny, and individuals do not always adhere passively to their inherited culture for the duration of their lives. We extend our model to analyze the case in which agents are "exposed" to other cultures, and are able to adopt a new culture.[4] Why would an agent adopt a restrictive culture when this inhibits their ability to achieve Pareto-efficient coordination? Adopting a *restrictive* culture (i.e. $X_k \subset X$) can commit a player to taking actions that she most prefers, but also entails miscoordination with adherents to other restrictive (and opposed) cultures. Adopting a *permissive* culture (i.e. $X_k = X$) on the other hand, enables better coordination with adherents to other cultures, but less ability to achieve coordination on the action profile that the individual most prefers. Therefore, adherence to a more restrictive culture plays the role of a strategic commitment in bargaining, whereby parties attempt to bolster their positions by limiting their options (Schelling 1960). To illustrate this point, consider the standard of dress example above. Suppose that, apart from coordination payoffs, all agents from community 1 most prefer dress $x_1 \in X$. By adopting a culture (i.e. action set) $X_1 = x_1$, community 1 agents commit to action x_1. As the choices of this community are observed over time, the community obtains a reputation for playing x_1.[5] Therefore, if members of community 2 want to coordinate, they need to adopt the standard of dress x_1, thus ensuring coordination on the most preferred action for community 1 members. However, if $x_1 \notin X_2$, then there is no possibility for coordination. Thus there are costs and benefits to choosing a restrictive culture, and it is not clear *a priori* whether restrictive cultures survive evolutionary selection.

The potential gains from coordination cannot be realized if miscoordination prevails in intercultural interactions. So play in the underlying coordination game coevolves with the distribution of players across cultures. Cultural choice, or analogously migration between cultural communities, is the mechanism for *cultural group selection* in our model. A culture dies out when it has no adherents. We analyze the size and composition of competing cultural communities in the long run, as individuals succeed and fail to achieve coordination in intercultural interactions. We uncover a surprising result that in a society which begins with three cultural communities, at most two cultures survive in the long run: one restrictive culture and one permissive culture.

[4]When an individual adopts a new culture it should be said that she is resocialized. Schaefer & Lamm (1992, p. 113) note that this is a frequent occurrence in the human life cycle.

[5]In our model, each culture has its own *marker*, and players observe the cultural marker of their partner prior to taking an action. According to Boyd & Richerson (2005): "Human populations are richly subdivided into groups marked by seemingly arbitrary symbolic traits, including distinctive styles of dress, cuisine or dialect. Such symbolically marked groups often have distinctive moral codes and norms of behavior, and sometimes exhibit economic specialization" [p. 99].

The remainder of the paper is structured as follows. Section 2, explicates the behavioral foundations of our approach to modeling culture. Section 3 sets out the version of our model without cultural choice, and Section 4 analyzes the evolution of play in intercultural interactions. In Section 5, we extend the model to analyze which cultural configurations survive in the long run when individuals can adopt a new culture.

2 Modeling Culture

2.1 Culture as Commitments

Culture derives from the Latin word meaning "to cultivate." The archetypal definition was provided by English Anthropologist Edward B. Tylor in his 1871 book, *Primitive Culture*: culture is "that complex whole which includes knowledge, belief, art, law, morals, custom, and any other capabilities and habits acquired by man as a member of society" (Tylor 1871, p.1). This all-encompassing concept of culture as all patterns of learned human behavior no doubt contributed to the 164 different definitions of culture listed by Kroeber & Kluckhohn (1952). We adhere in this paper to Boyd and Richerson's definition of culture: "Culture is information that people acquire from others by teaching, imitation and other forms of social learning" (Boyd & Richerson 2005, p.3). This information shapes beliefs and mental structures, which in turn get transformed into patterns of social and economic behavior (North 1993). For our purpose, a culture is identified by the particular patterns of behavior through which it gains expression.

We focus on one very specific aspect of culture that is nested in the foregoing definitions. In our model, culture generates *commitments*, or equivalently culture "limits choice sets" (Henrich et al. 2001, p. 357). Each culture proscribes a different set of actions, which can be thought of as cultural taboos. Individuals belonging to a particular cultural community must observe its taboos. Such cultural commitments have been largely ignored in prior analytical work. One way in which culture might generate commitments is by influencing what an individual believes is right and wrong, as Boyd and Richerson have noted. Bowles (2001) points out that "[t]o explain why a person chose a point in a budget set, for example, one might make reference to her craving for the chosen goods, or to a religious prohibition against the excluded goods" [p. 157]. So culture generates what Rao & Walton (2004) call "constraining preferences". Geertz (1973) claims that "[a]s the order of bases in a strand of DNA forms a coded program,…so culture patterns provide such programs for the institution of the social and psychological processes which shape public behavior" [p. 92], and that "[u]ndirected by culture patterns…man's behavior would be virtually ungovernable" [p. 46].

2.2 A Hybrid Model of Cultural Evolution

The formal analysis of cultural transmission via social contact was pioneered by Cavalli-Sforza & Feldman (1981) and Boyd & Richerson (1985, 2005). These social contagion models, including the agent-based model developed by Axelrod (1997), feature agents who do not engage in *intentional* choice, but are *passive* carriers of traits which are transmitted through social interaction like a virus (see also Coleman 1964, Bass 1969, Dodds & Watts 2005). We see an analogy between the acquisition of traits via social contact and socialization, which we believe is an important process in cultural evolution. On the other hand, intentional individual choice is an indispensable ingredient in models of social dynamics; agents are not really passive, even in the process of socialization. Dennis Wrong (1961) has argued persuasively against an "over-socialized" view of the individual, pointing out that while "socialization means transmission of culture, ... this does not mean that [individuals] have been *completely molded* by the particular norms and values of their culture." Therefore, we adopt the framework of *bounded rational choice* developed by Young (1993), and introduce to it a simple notion of culture. Coleman (1990) warns that "[t]o examine the process by which norms are internalized is to enter waters that are treacherous for a theory grounded in rational choice"; nevertheless "individual interests do change and individuals do internalize norms" [p.292-3]. In this essay, we develop a *hybrid* model in which agents adapt their behavior in an intentional manner, but within culturally accepted bounds of behavior which are internalized during socialization.

Bisin & Verdier (2001a) develop an overlapping-generations model of cultural transmission which combines socialization and rational choice. As in Cavalli-Sforza & Feldman (1981), agents are endowed with one of two cultural traits and each agent in the younger generation is exposed to a 'model' which is randomly selected from the older generation. Bisin & Verdier's innovation is to allow each parent to choose costly socialization effort τ (e.g. cultural education, segregation) in order the increase the chance that their offspring inherits their cultural trait. An agent retains their parent's trait with probability τ and acquires the trait borne by the model to whom they are exposed with the complementary probability. Bisin & Verdier show that the unique asymptotically stable distribution of traits is a polymorphic distribution, with positive weight on both traits. So by exerting greater socialization effort, minority cultures can survive the spread of the majority trait via social contagion. As with Bisin & Verdier, our model allows for intentionality in cultural choice. But whereas agents in Bisin & Verdier's model simply take a (non-strategic) decision based upon their cultural trait, we allow agents with different cultural traits to strategically interact by playing a coordination game. The evolution of play in the coordination game can make it profitable to switch cultures. In contrast, parents always prefer that their offspring inherit their cultural trait in Bisin & Verdier's formulation. The fact that multiple cultures can coexist in the long run in our model, is due to the benefits of cultural *commitment* on payoffs in the coordination game.[6]

[6] Axelrod (1997) develops an agent-based model in which multiple cultures can coexist in the long run. However, as he notes, this result is not robust to cultural mutations, in which players can randomly acquire mutant traits. In our model, multiple cultures can coexist even in the presence of mistakes and

As in this paper, Kuran & Sandholm (2008) develop a model in which social interactions take the form of a coordination game and agents have idiosyncratic preferences over actions. Agents in their model compromise between their ideal action and the action that yields the largest coordination payoff. However, culture in Kuran & Sandholm's model does not restrict an agent's choice set.[7] As such, there is no breakdown of Pareto-efficient coordination. Another point of difference is that Kuran & Sandholm assume preferences evolve so that an agent's ideal action converges to their actual action over time. In our model, individual preferences remain fixed and it is agents' choice sets – or equivalently the composition of cultural communities – which evolves.

3 The Model

3.1 Dyadic Interactions

Players. There are n *roles* that may be filled by a changing cast of players. Over time, we can think of players "dying" and their roles being filled by incoming players who inherit their predecessor's culture. This is analogous to the vertical transmission of culture from parent to chid. For convenience, we speak of n players (rather than roles), where $n \geq 2$ and typically large, but finite. The set of players is denoted by N, with members indexed by i.

Cultures. Each player belongs to one of $K \geq 2$ communities in any given period, denoted by $c \in C = \{c_1, c_2, ..., c_k, ..., c_K\}$. Each community has a different culture which is shared by its members. A player belonging to cultural community c_k is referred to as a c_k-member. The set of c_k-members is N_k, where $|N_k| = n_k$ and $\sum_k |N_k| = n$. A cultural community c_k is said to be *empty* if it has no members, i.e. $n_k = 0$. Each culture has its own visible *marker*, and individuals in a dyadic interaction observe each others' cultural markers *prior* to selecting an action.

Actions. The set X, with cardinality L, is the set of all *pure* strategies that can potentially be taken in a dyadic interaction. A player belonging to culture c_k must choose an action x from the set of pure strategies, X_k, which are acceptable to her culture, where $X_k \subseteq X$. This is the key assumption in our model:

Condition 1. *(Commitment)* A c_k-member in period t is *committed* to choosing from the set of actions, X_k, when interacting in period t.

Such a commitment might come about through the internalization of culturally accepted standards of behavior during socialization, or as a result of sanctions imposed by the cultural community, though we do not model this here. We do not assume that this commitment is common or even mutual knowledge. All that we require in this respect is that each player knows which actions they themselves are permitted to take

random experimentation on the part of agents. See also Bowles & Gintis (2004) on how ethnocentric networks can survive alongside an anonymous market offering unrestricted trading opportunities.

[7] Rather, culture is characterized by a distribution of preferences and actions among members of a cultural community.

in the current period. The following definitions enable us to characterize cultures via the subset of actions they proscribe:[8]

RESTRICTIVENESS. A culture c_k is *permissive* if $|X_k| = L$.

COMPATIBILITY. Two cultures c_j and c_k are *radically opposed* if $X_j \cap X_k = \emptyset$.

Payoffs. Players in a dyadic interaction benefit from coordinating their actions. In addition, individuals have idiosyncratic preferences over possible actions, that are independent of the action taken by their partner in a social interaction. Formally, payoffs in dyadic interactions are given by $u_i : X_j \times X_k \to \mathbb{R}$, for all $i \in N$ and $1 \leq j,k \leq K$. We assume u_i satisfies the usual von Neumann-Morgenstern axioms and takes the following form:

$$u_i(x, x_{-i}) = \mathscr{I} + \delta_{i,x}$$

Let $\mathscr{I}$ be player i's *interactive payoff*, where $\mathscr{I} = 1$ if $x = x_{-i}$ and zero otherwise. Therefore, the interactive payoff to each player is normalized to one for coordination, and zero for miscoordination. In addition, player i receives an *idiosyncratic payoff* equal to $\delta_{i,x}$ from playing action x. This idiosyncratic payoff represents player i's predisposition to taking certain actions. Payoffs in dyadic interactions are illustrated in Figure 1 by the payoff matrix for a 2×2 game.

Interactive Payoffs

	A	B
A	$1,1$	$0,0$
B	$0,0$	$1,1$

Idiosyncratic Payoffs

	A	B
A	$\delta_{1,A}, \delta_{2,A}$	$\delta_{1,A}, \delta_{2,B}$
B	$\delta_{1,B}, \delta_{2,A}$	$\delta_{1,B}, \delta_{2,B}$

Overall Payoffs

	A	B
A	$1+\delta_{1,A}, 1+\delta_{2,A}$	$\delta_{1,A}, \delta_{2,B}$
B	$\delta_{1,B}, \delta_{2,A}$	$1+\delta_{1,B}, 1+\delta_{2,B}$

Figure 1: Payoffs in a dyadic interaction which takes the form of a 2×2 game with action space {A,B}. Payoffs are decomposed into interactive and idiosyncratic components.

[8]More precisely, culture is information which shapes beliefs and mental structures, which in turn determine behavior. For our purpose, we simply identify a culture by the particular behavior through which it gains expression, and treat the intermediate stages in which beliefs and mental structures are formed as a black box. So in the interests of expositional convenience we at times speak loosely of culture as if it were an entity "proscribing actions" and occasionally equate culture with behavior.

We say that player i "most prefers" an action $\tilde{x}$, if $\tilde{x} \in argmax_{x \in X} \delta_{i,x}$. A player i most prefers an X_k action $\tilde{x}_k$, if $\tilde{x}_k \in argmax_{x \in X_k} \delta_{i,x}$. We treat an individual's idiosyncratic preferences as exogenous and private information to that individual. We impose the following non-degeneracy condition [ND] on idiosyncratic preferences:

Condition 2. *(Non-Degeneracy)* $|\delta_{i,x} - \delta_{i,x'}| < 1$ for all players i and for all actions $x, x' \in X$.

Due to players' idiosyncratic preferences, each dyadic interaction is a different game. When ND is violated, not all dyadic interactions take the form of a coordination game, since x is a strictly dominated strategy when $\delta_{i,x'} - \delta_{i,x} > 1$, for some $x \neq x'$. We are interested in social interactions that take the form of a coordination game, so we restrict our attention to preference distributions that satisfy ND. This mild condition is all we require on the distribution of preferences. When $\delta_{i,x} = 0$ for all players i and actions x, all dyadic interactions take the form of a pure coordination game. When individual preferences are heterogeneous and condition ND is satisfied, then dyadic interactions take the form of a coordination game with conflicting interests, exemplified by the *Battle of the Sexes*.

3.2 Adaptive Choice

Timing. Time is discrete and denoted by $t = 1, 2, 3, \ldots$ Each period, two players are selected to engage in a dyadic interaction. Each pair of players is selected with equal probability, regardless of the players' cultural affiliations.[9]

Information. A player selected to play a coordination game, forms an expectation of her partner's action in the social encounter, by looking back at the *history* of play. The player cannot isolate information on her partner's previous plays. However, she can observe her partner's cultural marker. Therefore, in our model players base their expectations on the prior plays of members of their partner's culture. We say that a *j-k interaction* occurs when a c_j-member and a c_k-member are paired to play a coordination game. We denote the history of play in a particular *j-k* interaction as follows:

HISTORIES. The *j-k history* in period t is denoted by $h^t_{j-k} = (\mathbf{x}^{t,-m}_{j-k}, \ldots, \mathbf{x}^{t,-1}_{j-k})$, which is the record of play in the m previous *j-k* interactions. Play in the most recent *j-k* interaction is captured by the action-tuple $\mathbf{x}^{t,-1}_{j-k} = (x^{t,-1}_{j,k}, x^{t,-1}_{k,j})$.

The individual action denoted by $x^{t,-1}_{j,k}$ is at time t the most recent action taken by a c_j-member in a *j-k* interaction, and $x^{t,-m}_{j,k}$ is the m^{th} most recent such action. In a *k-k* interaction, in which members of the same culture are paired, players are randomly allocated to the "first player" and "second player" positions, where the action taken by the first player is recorded as the first element of $\mathbf{x}^t_{k-k}$.[10] The history of play at time

[9]Therefore, cultural membership does not define a local interactions structure in which individuals are more likely to interact with adherents to their own culture.

[10]By "random", we mean that the draw is from a distribution with full support on the relevant set.

t, which is denoted by the vector $h^t = (h^t_{j-k})_{j \leq k}$, is a record of the previous m action-tuples played in every j-k interaction. This is not the same as in the canonical model (Young 1993), in which agents remember the actions played in the last m periods. If we were to retain the standard formulation, a particular j-k interaction could occur in m consecutive periods, leaving the j'-k' histories empty, for all $j' \neq j$ and $k' \neq k$.[11]

When forming expectations, players in a j-k interaction obtain fragmented information on plays in prior j-k interactions by sampling the j-k history. In concrete terms, we can think of agents asking around about the experiences of players in earlier periods.[12] We formalize this as follows. In a j-k interaction which occurs in period t, the c_k-member draws a sample of size s (without replacement) from the j-k history h^t_{j-k}, and specifically from the last m actions taken by c_j-members in interactions with c_k-members. The c_j-member does the same, except that she samples from the actions taken by c_k-members in the m previous j-k interactions.[13] Apart from mistakes, both players independently choose (myopic) best replies to their resulting sample proportions, as illustrated below.

Social Behavior. Agents are boundedly rational, in the sense that they *myopically* maximize their expected *current* period payoff when choosing an action in a dyadic interaction. The c_k-member in a j-k interaction forms her expectation after drawing a sample of size s from the j-k history, h^t_{j-k}. This occurs with high probability $(1 - \varepsilon)$. The agent then calculates the proportion, $\widehat{p}_{j,k}(x)$, of c_j-members playing action x in j-k interactions, for each $x \in X_j$. The c_k-member adopts this as a maximum likelihood estimate of the behavioral strategy used by a c_j-member in a j-k interaction. Therefore, the expected payoff to the c_k-member i from playing action x in a j-k interaction is $\widehat{p}_{j,k}(x) + \delta_{i,x}$. This in turn yields the set of pure strategies, $argmax_{x \in X_k}\{\widehat{p}_{j,k}(x) + \delta_{i,x}\}$, which maximize player i's expected current period payoff, given her sample information. When there are ties in best replies, each best reply is played with equal probability. With low probability ε, the c_k-member (resp. c_j-member) instead chooses an action in X_k (resp. X_j) at random, for reasons outside of the model.

Coordination and Miscoordination. A convention is a strict Nash equilibrium that has been played by the entire population for as long as anyone can remember (Young 1993). This means that any possible sample drawn from the history of play will be identical, and yield the same best reply, namely the conventional action itself. Let $\mathbf{x}^*_{j-k}$ be a strict Nash (coordination) equilibrium of a coordination game between c_j-

[11]According to our formulation, play in a j-k interaction in period t' may be part of the period t history, while play in a j'-k' interaction in period $t'' > t'$ has been forgotten, if more j'-k' interactions occur in recent periods. We do not consider this to be implausible. For example, a person might remember her last encounter with an old friend which occurred years ago, while forgetting a more recent encounter with a friend she meets weekly. Our approach also ensures the independence of play in each j-k interaction.

[12]It should be clear, however, that we do not model this as an optimal search process, but rather the agent's information is considered to be a property of her environment.

[13]In a k-k interaction we assume that a player allocated to the first (resp. second) player position, randomly samples s of the m most recent plays by individuals occupying the second (resp. first) player position.

members and c_k-members, where $\mathbf{x}^*_{j-k}$ indicates that $x_{j,k} = x_{k,j} \in X_j \cap X_k$.[14] We define a convention as follows:

CONVENTION. A *convention* is a history h^*_{j-k} in which a strict Nash equilibrium $\mathbf{x}^*_{j-k}$ is played in m consecutive j-k interactions. We say the *j-k convention* is "x" when $h^*_{j-k} = (\mathbf{x}^*_{j-k}, \mathbf{x}^*_{j-k}, \ldots, \mathbf{x}^*_{j-k}) = ((x,x),(x,x),\ldots,(x,x))$, where $x \in X_j \cap X_k$.

When cultures c_j and c_k are radically opposed, there are no *mutually admissible* actions that players can take, i.e. $X_j \cap X_k = \emptyset$, so a convention cannot arise, and players in a j-k interaction are doomed to perpetual miscoordination. We characterize such a state of miscoordination as follows:

STATE OF MISCOORDINATION. A *state of miscoordination* is a j-k history $h^{mc}_{j-k} = ((x^1_j, x^1_k), \ldots, (x^\ell_j, x^\ell_k), \ldots, (x^m_j, x^m_k))$ in which $x^\ell_j \notin X_k$ and $x^\ell_k \notin X_j$ for all $1 \le \ell \le m$.

3.3 Dynamical Process

As individuals interact recurrently, their adaptive behavior gives rise to a particular dynamical process at the population level, which we will now characterize. The state in period t, $z^t = (h^t)$, specifies the history of play in period t. The associated state space is $Z = \prod_{j \le k} (X_j \times X_k)^m$, where $(X)^m$ denotes the m-fold product of X. Clearly, Z is finite for finite K and m. We assume that the process begins in some initial state z^0 in which there are at least m plays in each j-k interaction, where the sequence of plays is otherwise arbitrary. There are well-defined, time-homogeneous transition probabilities between all pairs of states z, z', denoted by $P_{z,z'}$. Therefore, the adaptive process we have defined is a finite Markov chain, with a $|Z| \times |Z|$ transition probability matrix $P^{m,s,\varepsilon}$. For convenience, we will denote the unperturbed ($\varepsilon = 0$) process by P^0 and the perturbed process by P^ε. P^ε is a regular perturbed Markov process (Young, 1993). When $\varepsilon > 0$, all pairs of states communicate, so the process P^ε is irreducible. This implies that the process has a unique recurrence class, the entire state space, so the Markov chain is ergodic, i.e. its limiting distribution is independent of the initial state z^0. Moreover, the Markov process P^ε has a unique stationary distribution μ^ε. The perturbed process is aperiodic, since there is a positive probability of remaining in any given state. This implies that not only does the relative frequency with which a state z is visited *up through* time t converge to the frequency given by the unique stationary distribution μ, but so does the probability of being in state z *at* time t, provided that t is sufficiently large. We rely on the following equilibrium concept due to Foster & Young (1990):

STOCHASTIC STABILITY. A state is *stochastically stable* if it is in the support of $\mu = lim_{\varepsilon \to 0} \mu^\varepsilon$.

[14] Notice that even though idiosyncratic preferences mean each dyadic interaction is a different game, condition ND ensures that the strict Nash equilibria of this game are the same for all dyadic interactions between c_j-members and c_k-members.

4 Coordination and Miscoordination

What are the prospects for coordination in social interactions when individuals adhere to different cultures? In this section, we focus on the following case: (i) there are two cultures c_1 and c_2, and dyadic interactions are always 1-2 interactions in which a c_1-member and a c_2-member are matched, (ii) there is no migration between cultural communities. In this case, we recover a process that is similar to Young's (1993) *adaptive play*, except that certain actions are proscribed by a player's culture.[15]

4.1 The Possibility of Long-Run Miscoordination

This subsection analyzes the long-run behavior of the *unperturbed* Markov process. We denote by P^0_{1-2} the 1-2 process under conditions (i) and (ii). The state of P^0_{1-2} in period t is fully characterized by the 1-2 history in that period, that is $z^t = h^t_{1-2}$. The process operates on the finite state space H_{1-2}, which is the set of all possible 1-2 histories, defined as follows: $H_{1-2} = (X_1 \times X_2)^m$, where $(X)^m$ denotes the m-fold product of X.

By studying the behavior of P^0_{1-2}, we can isolate the effect on the evolution of play of (a) cultural restrictions on individual behavior, and (b) substantial heterogeneity in individual preferences. In Young's (1993) adaptive play, coordination is always achieved in the long run. In a moment, we will show that social conventions can emerge despite cultural restrictions and substantial heterogeneity in agents' preferences. However, the introduction of cultural restrictions yields a new long-run possibility, in which coordination permanently breaks down, even when cultures c_1 and c_2 are not radically opposed. It will be straightforward to generalize these results to a society with $K > 2$ cultures (see Section 5).

First, we need to introduce some definitions. A c_k-member i maximizes her idiosyncratic payoff, without regard for coordination, if she simply chooses her *most preferred* X_k action $\tilde{x}_{i,k} \in argmax_{x \in X_k} \delta_{i,x}$. Let $\tilde{X}_k = \{\tilde{x} \mid \tilde{x} \in argmax_{x \in X_k} \delta_{i,x}$ for some $i \in N_k\} \subseteq X_k$. We can now define a further pairwise relation on the set of cultures:

WEAK OPPOSITION. Cultures c_j and c_k are *weakly opposed* if and only if $\tilde{X}_j \cap X_k = \emptyset$ and $\tilde{X}_k \cap X_j = \emptyset$.

If cultures c_j and c_k are radically opposed, i.e. $X_j \cap X_k = \emptyset$, then they are also weakly opposed, since $\tilde{X}_j \subseteq X_j$ and $\tilde{X}_k \subseteq X_k$. However, weak opposition unlike radical opposition depends on the distribution of idiosyncratic preferences in a cultural community.

Define the set $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$. When c_1 and c_2 are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M}$ is a *state of miscoordination* (see section 3.2). Recall that a recurrence class of a Markov process is a set of states, each of which is

[15] We will show (in a corollary) that our results apply to the case in which players from the same culture are matched in a dyadic interaction. This yields one further departure from Young's approach. In our model, players in a *k-k* interaction are drawn from a single population, whereas in Young's version they are always drawn from separate populations.

accessible from any other state within the class, and for which no state outside the class is accessible from any state within the class.[16] In a moment, we will show that $\mathcal{M}$ is the *unique* recurrence class of P^0_{1-2}, when c_1 and c_2 are radically opposed. Since all states in $\mathcal{M}$ are states of miscoordination when the cultures are at least weakly opposed, and the process can become "locked into" $\mathcal{M}$, we refer to this long-run possibility as *recurrent miscoordination*. That recurrent miscoordination arises when cultures are radically opposed is not very surprising. Indeed, we will show that when cultures c_1 and c_2 are not radically opposed, the picture is somewhat more sanguine regarding coordination, in that the conventions are recurrence classes of P^0_{1-2}, despite cultural restrictions on behavior. However, the striking result is that even when the cultures are only *weakly* opposed, recurrent miscoordination $\mathcal{M}$ remains a recurrence class. We have the following result:

Theorem 1 *Suppose $s/m \leq 1/2$. Then P^0_{1-2} converges almost surely to: (i) a convention h_1^*, h_2^*, ... or h_L^*, if the cultures are not weakly opposed, (ii) a convention or recurrent miscoordination $\mathcal{M}$ if the cultures are weakly opposed, but not radically opposed, (iii) $\mathcal{M}$ if the cultures are radically opposed.*

Proof. Follows directly from Lemmas 1 and 2, below.

The proof of Theorem 1 proceeds in two steps. First, we identify the conditions under which the conventions and the set $\mathcal{M}$ are recurrence classes of P^0_{1-2}. We then show that if $s/m \leq 1/2$, these are the only recurrence classes of the process.[17] Any finite Markov chain such as P^0_{1-2} converges almost surely to one of its recurrence classes. So this suffices to establish the Theorem.

Lemma 1 *(i) The conventions are recurrence classes of P^0_{1-2} if and only if the cultures are not radically opposed, (ii) $\mathcal{M}$ is a recurrence class if and only if the cultures are weakly opposed.*

Proof. To establish part (i), recall that a best reply for a c_1-member is an action $x^* \in argmax_{x \in X_1}\{\widehat{p}_{2,1}(x) + \delta_{i,x}\}$. A convention is a state of the form $((x,x),(x,x),\ldots(x,x))$, where $x \in X_1 \cap X_2$. So a convention can only arise if $X_1 \cap X_2 \neq \emptyset$, that is when c_1 and c_2 are not radically opposed. Without loss of generality, suppose an x_1 convention is in place. Then for all possible samples from h_1^* drawn by a c_1-member, the sample proportion of x_1 plays is $\widehat{p}_{2,1}(x_1) = 1$ and the sample proportion of x plays is $\widehat{p}_{2,1}(x) = 0$ for all $x \neq x_1$. So the expected payoff from playing action x_1 to a representative c_1-member i, for all possible samples when an x_1 convention is in place, is $\widehat{p}_{2,1}(x_1) + \delta_{i,x_1} = 1 + \delta_{i,x_1}$. The expected payoff from playing any action $x \neq x_1$ is $\delta_{i,x}$. According to condition ND, $|\delta_{i,x} - \delta_{i,x'}| < 1$ for all players i and for all actions

[16] Also recall that a state z is accessible from state z', if there is a positive probability of moving from state z to z' in a finite number of periods.

[17] By making s/m sufficiently small, we ensure that there is enough randomness in the system to shake the process out of any other possible cycle. We do not claim that the bound $s/m \leq 1/2$ is the tightest possible bound for all preference distributions, only that it is sufficient to deliver the result without placing further restrictions on preferences.

$x, x' \in X$. This implies that $1 + \delta_{i,x_1} > \delta_{i,x}$ for all i and x. Therefore, the unique best reply for any $i \in N$ is always x_1 when an x_1 convention is in place. In the unperturbed case ($\varepsilon = 0$) we are considering, there is a zero probability that a non-best reply is played. So no matter which players are drawn, the successor state to a convention $((x_1, x_1), (x_1, x_1), \ldots (x_1, x_1))$ is certainly $((x_1, x_1), (x_1, x_1), \ldots (x_1, x_1))$, and each convention is an absorbing state (i.e. a singleton recurrence class).

We shall now establish part (ii). When the cultures are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$ is a state of miscoordination. Now consider such a state in $\mathcal{M}$. For all possible samples drawn by c_1-member from a state of miscoordination, the sample proportion of action x is zero for all $x \in X_1$, since all plays by c_2-members in such a history are not in X_1. Therefore, the best reply for all c_1-members to a state of miscoordination is always to choose an action $\tilde{x}_{i,1} \in argmax_{x \in X_1} \delta_{i,x} \subseteq \tilde{X}_1$. Similarly, the best reply for all c_2-members to a state of miscoordination is always to choose an action $\tilde{x}_{i,2} \in argmax_{x \in X_2} \delta_{i,x} \subseteq \tilde{X}_2$. So once a state of miscoordination is reached, the action-tuple played in the next period is certainly in $\tilde{X}_1 \times \tilde{X}_2$, and can be any pair in $\tilde{X}_1 \times \tilde{X}_2$. This implies that no state outside $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$ is accessible from any state in $\mathcal{M}$. Furthermore, every state in $\mathcal{M}$ is accessible from every other state in $\mathcal{M}$. Hence $\mathcal{M}$ is a recurrence class of P^0_{1-2} if c_1 and c_2 are weakly opposed. Notice that if the cultures are not weakly opposed, then there exists an action $x' \in \tilde{X}_1 \cap X_2$. This implies that $((x', x), (x', x), \ldots (x', x))$ is in $\mathcal{M}$ for all $x \in \tilde{X}_2$. But all possible samples from such a state which can be drawn by a c_2-member, consist of s plays of $x' \in X_1 \cap X_2$. It follows from the proof of Lemma 2, case 1 (see the Appendix), that P^0_{1-2} transits from this point to the x' convention with positive probability in at most another m periods. By part (i) of Lemma 1, each convention is an absorbing state. This means that no other state in $\mathcal{M}$ is accessible from a convention. Therefore, if c_1 and c_2 are not weakly opposed, the set $\mathcal{M}$ is not a recurrence class, unless it is simply the x' convention. □

To understand why recurrent miscoordination is a recurrence class when c_1 and c_2 are weakly opposed, suppose that the process is in a state of miscoordination. For all possible samples, both individuals selected to play expect that the probability of coordinating is zero. As far as they know, c_1-members have always taken actions that are inadmissible for c_2-members, and *vice versa*. Therefore, both individuals play one of their most preferred actions. If c_1 and c_2 are weakly opposed, then no most preferred action is in $X_1 \cap X_2$, so the process remains in a state of miscoordination. If the cultures are not weakly opposed, then at least one player will take a mutually admissible action in $X_1 \cap X_2$, so the unperturbed process can exit from a state of miscoordination, and a convention can emerge.

In conjunction with Lemma 1, the following suffices to establish Theorem 1:

Lemma 2 *If $s/m \leq 1/2$, then a set of states $\mathscr{X} \subset Z$ is a recurrence class of P^0_{1-2} only if it is a convention or $\mathcal{M}$.*

The proof is in the Appendix. Theorem 1 fully characterizes the asymptotic behavior of P^0_{1-2}. Firstly, conventions can emerge in intercultural interactions despite cultural restrictions on behavior, even when the cultures are weakly opposed. Secondly, we have derived the results with only a mild condition (ND) on the distribution of preferences. Therefore, our analysis incorporates a far greater degree of heterogeneity in individual preferences than does prior work. Young (1998) studies 2×2 coordination games in which preferences can vary between disjoint classes of players, but players within each disjoint class have the same preferences. Our analysis goes substantially further by allowing every individual to have different preferences. Thirdly, we have uncovered an interesting long-run possibility that has not been identified in previous work. Coordination can *permanently* break down in intercultural interactions even where it need not, and even though coordination is *Pareto-efficient*.

The risk of "clashes" between members of disparate cultural communities has been emphasized by several influential commentators (e.g. Huntington 1993). We have shown how an analogous situation can arise within the framework of a (modified) coordination game. Since we partition society on the basis of culture alone, this result can also be viewed as support for Sen's (2006) claim that the "clash of civilizations" thesis follows naturally from a singular concept of identity. In our model, permanent miscoordination can arise after a string of failed social interactions, which leads c_1-members to lose confidence that c_2-members will play a mutually admissible action, and *vice versa*. This can happen even when $X_1 \cap X_2 \neq \emptyset$, so that coordination can be achieved within cultural restrictions. Nevertheless, if initially all players are confident enough of achieving coordination, a convention might emerge which coordinates behavior in social interactions.

The analysis so far has been conducted for interactions between members of two different cultures c_1 and c_2. We now derive the corresponding result for the evolution of play in interactions between adherents to the same culture.

Corollary 1 *Suppose $s/m \leq 1/2$. Then P^0_{k-k} converges almost surely to a convention.*

The proof is in the Appendix. Corollary 1 shows that a recurrent miscoordination cannot arise in interactions between adherents to the same culture, for *any* distribution of individual preferences. We attribute the fact that players share the same set of admissible actions to shared beliefs regarding acceptable behavior. Therefore, this result captures the intuition that coordination in social interactions is more likely when individuals have the same cultural beliefs.

4.2 Stochastically Stable Miscoordination

We have established that coordination between members of cultural communities which are weakly opposed can permanently break down. However, as long as the cultures are not radically opposed, the conventions remain recurrence classes of P^0_{1-2}. So we might expect that recurrent miscoordination is a tenuous phenomenon when cultures c_1 and c_2 are not radically opposed. In this section, we employ the stochastic stability framework to show that, on the contrary, there exist a range of distributions of individual preferences for which *recurrent miscoordination is the most likely outcome of play in the long run.*

For this purpose, we introduce the possibility that players make a mistake or engage in random experimentation with probability $\varepsilon > 0$. This gives rise to the perturbed process P^{ε}_{1-2}. Taking the limit $\varepsilon \to 0$ allows us to make sharp statements about the asymptotic behavior of P^{ε}_{1-2}. We will also make use of two new conditions on the distribution of individual preferences. Assume $X_1 \cap X_2 = \{x_1, \ldots x_\ell, \ldots x_L\}$. Define:

$$\underline{\delta}_{1,\ell} = min_{i \in N_1}(max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

Intuitively, $\underline{\delta}_{1,\ell}$ represents the *weakest* idiosyncratic preference for miscoordination over coordination on x_ℓ, among c_1-members. Similarly, define $\underline{\delta}_{2,\ell} = min_{i \in N_2}(max_{x' \in X_2 \setminus X_1} \delta_{i,x'} - \delta_{i,x_\ell})$. Let $\underline{\delta}_\ell = min\{\underline{\delta}_{1,\ell}, \underline{\delta}_{2,\ell}\}$ represent the *weakest* idiosyncratic preference for miscoordination over coordination on x_ℓ, among all players. Then $\underline{\delta} = min_{1 \leq \ell \leq L} \underline{\delta}_\ell$ represents the *weakest* idiosyncratic preference for miscoordination. Notice that c_1 and c_2 are weakly opposed if and only if $\underline{\delta} > 0$. Now let:

$$\overline{\delta}_{1,\ell} = max_{i \in N_1}(max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

Define $\overline{\delta}_{2,\ell}$ similarly. Also, let $\overline{\delta}_\ell = max\{\overline{\delta}_{1,\ell}, \overline{\delta}_{2,\ell}\}$. Then $\overline{\delta} = min_{1 \leq \ell \leq L} \overline{\delta}_\ell$ represents the *strongest* idiosyncratic preference for miscoordination. To see this suppose that $\underline{\delta}_{1,\ell} < \underline{\delta}_{2,\ell}$ and $\overline{\delta}_{1,\ell} > \overline{\delta}_{2,\ell}$ for all ℓ, $1 \leq \ell \leq L$. In this case:

$$\underline{\delta} = min_{x_{\ell \in X_1 \cap X_2}} min_{i \in N_1}(max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

$$\overline{\delta} = min_{x_{\ell \in X_1 \cap X_2}} max_{i \in N_1}(max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

We now demonstrate that when the *weakest* and *strongest* idiosyncratic preference for miscoordination are sufficiently intense, then miscoordination is the most likely outcome of play:

Theorem 2 *Suppose c_1 and c_2 are weakly opposed and $s/m \leq 1/2$. If $\underline{\delta} + \overline{\delta} > 1$, then for s (and m) sufficiently large, recurrent miscoordination $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}.*

Proof. By Theorem 1, when c_1 and c_2 are radically opposed, $\mathcal{M}$ is the unique recurrence class of P^0_{1-2}. This directly implies that $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}, in this case. Now assume that c_1 and c_2 are not radically opposed. In this case, $X_1 \cap X_2 \neq \emptyset$ and we write $X_1 \cap X_2 = (x_1, \ldots x_\ell, \ldots x_L)$. In addition, assume that $s/m \leq 1/2$, and c_1 and c_2 are weakly opposed. By Theorem 1 part (ii), the only recurrence classes of P^0_{1-2} are the conventions $h^*_1, h^*_2, \ldots h^*_L$ and the recurrent miscoordination $\mathcal{M}$. So the stochastically stable classes are among these (Young 1993). The perturbed process P^{ε}_{1-2} can transit from a state in one recurrence class to a state in another, with a sequence of erroneous plays. Define the *resistance* $r(z, \mathcal{M})$ of the transition $z \to \mathcal{M}$ as the minimum number of errors required for the process to transit from state z to a state in $\mathcal{M}$.

To prove the Theorem, we employ the *radius-coradius* technique of Ellison (2000). In our model, the *radius* of the *basin of attraction* of $\mathcal{M}$, $R(\mathcal{M})$, is the minimum resistance of a path between a state in $\mathcal{M}$ and a convention, or formally:

$$R(\mathcal{M}) = min_{1 \leq \ell \leq L} r(\mathcal{M}, h^*_\ell)$$

The *coradius* of the basin of attraction of $\mathcal{M}$, $CR(\mathcal{M})$, is the maximum resistance of a path between a convention and a state in $\mathcal{M}$:

$$CR(\mathcal{M}) = max_{1 \leq \ell \leq L} r(h^*_\ell, \mathcal{M})$$

We claim that $R(\mathcal{M}) > CR(\mathcal{M})$ if $\underline{\delta} + \overline{\delta} > 1$, for s (and m) are sufficiently large. By Ellison (2000) Theorem 1, if $R(\mathcal{M}) > CR(\mathcal{M})$ then $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}. So this would suffice to establish the Theorem.

To verify the claim, we first compute $R(\mathcal{M})$. Consider a transition between a state in $\mathcal{M}$ and h^*_ℓ. Suppose that in period t, P^{ε}_{1-2} is in a state in $\mathcal{M}$. Since c_1 and c_2 are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M} = (\tilde{X}_2 \times \tilde{X}_2)^m$ is a state of miscoordination. For all period t samples then, the sample proportion of any action $x \in X_1 \cap X_2$ is zero. Fix ℓ. Now suppose the c_2-member(s) drawn *erroneously* play action $x_\ell \in X_1 \cap X_2$ in the next b consecutive periods. Then in period $t + b + 1$, there is a positive probability that player $i \in N_1$ draws a sample of size s, which contains all b plays of x_ℓ. Based on this sample, i's expected payoff from playing action x_ℓ is $b/s + \delta_{i,x_\ell}$. Player i's expected payoff from an action $x' \in X_1 \backslash X_2$ is always $\delta_{i,x'}$. Since the sample proportion of any action $x \in (X_1 \cap X_2) \backslash x_\ell$ is zero, the expected payoff to i from any action $x \in (X_1 \cap X_2) \backslash x_\ell$, is $\delta_{i,x}$. Because c_1 and c_2 are weakly opposed, $argmax_{x \in X_1} \delta_{i,x}$ is contained in $X_1 \backslash X_2$ for all $i \in N_1$. Therefore, no action $x \in (X_1 \cap X_2) \backslash x_\ell$ is a best reply. Action x_ℓ is then a best reply for i, for a sample including all b erroneous plays, if $b/s + \delta_{i,x_\ell} \geq \delta_{i,x'}$ for all $x' \in X_1 \backslash X_2$, or equivalently if:

$$b \geq max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \tag{1}$$

By (1), the minimum number of errors required for $x_\ell \in X_1 \cap X_2$ to be a best reply for i is:

$$b_i = \lceil max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \rceil \tag{2}$$

Therefore, given b_i erroneous plays by a c_2-member, there is a positive probability that $i \in N_1$ who is selected to play in period $t + b_i + 1$, samples all b_i errors, and plays a best reply $x_\ell \in X_1 \cap X_2$ to this sample. There is also a positive probability that i is selected to play in the next s consecutive periods, draws the same sample on each occasion (since $s/m \leq 1/2$), and plays the same best reply $x_\ell \in X_1 \cap X_2$ every time. It follows directly from the proof of Lemma 2, case 1 (see Appendix), that from this point the process transits to h^*_ℓ with no further errors.

The minimum number of errors required to transit from a state in $\mathcal{M}$ to h^*_ℓ, across all $i \in N_1$, is $b^*_{1,\ell} = min_{i \in N_1} \lceil max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \rceil = \lceil min_{i \in N_1} max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \rceil$. Similarly, the minimum number of errors for the transition $\mathcal{M} \to h^*_\ell$, across all $i \in N_2$, is $b^*_{2,\ell} = \lceil min_{i \in N_2} max_{x' \in X_2 \backslash X_1} (\delta_{i,x'} - \delta_{i,x_\ell}) s \rceil$. Without loss of generality, assume $b^*_{1,\ell} < b^*_{2,\ell}$ for all ℓ, $1 \leq \ell \leq L$, so that $b^*_{1,\ell}$ is the resistance of the transition $\mathcal{M} \to h^*_\ell$. $R(\mathcal{M})$ is the minimum such resistance over all ℓ:

$$\begin{aligned} R(\mathcal{M}) &= min_{x_{\ell \in X_1 \cap X_2}} \lceil min_{i \in N_1} max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \rceil \\ &= \lceil min_{x_{\ell \in X_1 \cap X_2}} min_{i \in N_1} (max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}) s \rceil \\ &= \lceil \underline{\delta} s \rceil \end{aligned} \tag{3}$$

We shall now establish an upper bound on $CR(\mathcal{M})$. Fix ℓ, and consider a transition between h^*_ℓ and a state in $\mathcal{M}$. Suppose that in period t, P^ε_{1-2} is in the x_ℓ convention $((x_\ell, x_\ell), \ldots (x_\ell, x_\ell))$. For all possible period t samples, the sample proportion of action x_ℓ is one. Now suppose the c_2-member(s) drawn erroneously plays an action $x'' \in X_2 \backslash X_1$ in the next $b \leq s$ consecutive periods. Then in period $t + b + 1$, there is a positive probability that player $i \in N_1$ draws a sample of size s, which contains all b plays of x''. Based on this sample, i's expected payoff from playing action x_ℓ is $(1 - b/s) + \delta_{i,x_\ell}$. Player i's expected payoff from an action $x' \in X_1 \backslash x_\ell$ is $\delta_{i,x'}$. So the only possible best replies for i in period $t + b + 1$ are x_ℓ or $\tilde{x} \in argmax_{x \in X_1} \delta_{i,x}$. [By hypothesis c_1 and c_2 are weakly opposed, so $\tilde{x} \notin X_1 \cap X_2$.] Action $\tilde{x}$ yields a weakly higher payoff for i than x_ℓ, for a sample including all b erroneous plays, if $\delta_{i,\tilde{x}} \geq (1 - b/s) + \delta_{i,x_\ell}$, or equivalently if:

$$b \geq (1 - (\delta_{i,\tilde{x}} - \delta_{i,x_\ell})) s \tag{4}$$

By (4), the minimum number of errors required for $\tilde{x} \in X_1 \backslash X_2$ to be a best reply for i is:

$$b_i = \lceil (1 - (\delta_{i,\tilde{x}} - \delta_{i,x_\ell})) s \rceil = \lceil (1 - max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})) s \rceil \tag{5}$$

The second equality follows from the definition of $\tilde{x}$. Therefore, given b_i or more erroneous plays by a c_2-member, there is a positive probability that $i \in N_1$ who is selected to play in period $t + b_i + 1$, samples all b_i errors, and plays a best reply $\tilde{x} \in X_1 \backslash X_2$ to this sample. There is also a positive probability that i is selected to play in the next s consecutive periods, draws the same sample on each occasion (since $s/m \leq 1/2$), and plays the same best reply $\tilde{x}$ every time. Since $\tilde{x} \notin X_1 \cap X_2$ and c_1 and c_2 are weakly opposed, we are in case 2(b) considered in the proof of Lemma 2. It follows that from this point, the unperturbed process P^0_{1-2} transits to a state in $\mathcal{M}$ with positive probability in at most $s + m$ additional periods. Therefore, P^ε_{1-2} can with b_i errors only, transit from h^*_ℓ to a state in $\mathcal{M}$.

The minimum number of errors across all $i \in N_1$ required to transit from h^*_ℓ to a state in $\mathcal{M}$ is at most $b^{**}_{1,\ell} = min_{i \in N_1} \lceil (1 - max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})) s \rceil = \lceil (1 - max_{i \in N_1} (max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell})) s \rceil$. Similarly, the minimum number of errors for the transition $h^*_\ell \to \mathcal{M}$, across all $i \in N_2$, is at most $b^{**}_{2,\ell} = \lceil (1 - max_{i \in N_2} (max_{x' \in X_2 \backslash X_1} \delta_{i,x'} -$

$\delta_{i,x_\ell}))s\rceil$. Without loss of generality, assume $b^{**}_{1,\ell} < b^{**}_{2,\ell}$ for all ℓ, $1 \leq \ell \leq L$, so that $b^{**}_{1,\ell}$ is an upper bound on the resistance of the transition $h^*_\ell \to \mathcal{M}$. $CR(\mathcal{M})$ is the maximum such resistance over all ℓ. Therefore:

$$\begin{aligned} CR(\mathcal{M}) &\leq max_{x_{\ell \in X_1 \cap X_2}} \lceil (1 - max_{i \in N_1}(max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil \\ &= \lceil (1 - min_{x_{\ell \in X_1 \cap X_2}} max_{i \in N_1}(max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil \\ &= \lceil (1 - \overline{\delta})s \rceil \end{aligned} \tag{6}$$

So far we have established that $R(\mathcal{M}) = \lceil \underline{\delta} s \rceil \geq \underline{\delta} s$, and that $CR(\mathcal{M}) \leq \lceil (1-\overline{\delta})s \rceil \leq (1-\overline{\delta})s + 1$. Therefore, $R(\mathcal{M}) > CR(\mathcal{M})$ if $\underline{\delta} s > (1-\overline{\delta})s + 1$ or equivalently if:

$$\underline{\delta} + \overline{\delta} > 1 + \frac{1}{s} \tag{7}$$

By observation, if $\underline{\delta} + \overline{\delta} > 1$, there exists an integer $\bar{s}$ such that inequality (7) is satisfied for all $s \geq \bar{s}$. This establishes the claim, and concludes the proof. $\square$

The intuition behind the result is as follows. It is more difficult for the process P^ε_{1-2} to exit the basin of attraction of recurrent miscoordination $\mathcal{M}$ when the weakest idiosyncratic preference for miscoordination, $\underline{\delta}$, is high. At the same time, it is relatively easy for P^ε_{1-2} to enter the basin of attraction of $\mathcal{M}$ when the strongest idiosyncratic preference for miscoordination, $\overline{\delta}$, is high. Therefore, when $\underline{\delta}$ and $\overline{\delta}$ are sufficiently high, it is difficult to exit the basin of attraction of $\mathcal{M}$ and easy to enter it. So as $\varepsilon \to 0$, P^ε_{1-2} spends virtually all of the time as $t \to \infty$ in $\mathcal{M}$, regardless of where the process begins. We have demonstrated that if c_1 and c_2 are weakly opposed, and s (and m) are sufficiently large, then $\underline{\delta} + \overline{\delta} > 1$ guarantees that individuals miscoordinate virtually all of the time. We remark that these conditions are not necessary for recurrent miscoordination to be stochastically stable. Nevertheless, they are stronger than weak opposition alone (i.e. $\underline{\delta} > 0$). The following "symmetric" example illustrates that the condition $\underline{\delta} + \overline{\delta} > 1$ can hold in plausible cases:

Example 1 *All individuals have the same preferences over actions, where $\delta_{i,x} = \delta_{\mathcal{M}}$ for all $(i,x) \in (N_1 \times X_1 \backslash X_2) \cup (N_2 \times X_2 \backslash X_1)$, and $\delta_{i,x} = \delta_C$ for all $(i,x) \in (N \times X_1 \cap X_2)$.*

The cultures c_1 and c_2 are weakly opposed if $\delta_{\mathcal{M}} > \delta_C$. In this case, $\mathcal{M}$ is a recurrence class of P^0_{1-2}, by Theorem 1. The "symmetry" of preferences implies that the strongest and weakest idiosyncratic preferences for miscoordination are the same, that is $\underline{\delta} = \overline{\delta} = \delta_{\mathcal{M}} - \delta_C$. Therefore, by inequality (7), $\mathcal{M}$ is the unique stochastically stable class if:

$$\delta_{\mathcal{M}} - \delta_C > \frac{1}{2}\left(1 + \frac{1}{s}\right) \tag{8}$$

The right hand side of (8) goes to $\frac{1}{2}$ as $s \to \infty$. Therefore, for s sufficiently large, $\mathcal{M}$ is the unique stochastically stable class of P^ε_{1-2}, if the idiosyncratic preference for miscoordination, $\delta_{\mathcal{M}} - \delta_C$, is greater than half the coordination payoff.

5 Cultural Evolution via Cultural Choice

When individuals make a reasoned choice of culture, which cultures survive in the long run, and which cultures die out? To answer this question, we extend our model to incorporate cultural choice. In this more general case, play in dyadic interactions *coevolves* with the composition of cultural communities. Our focus here is on a salient case called a *tripartite society*, in which there are three cultures. We show that at least one culture dies out in the long run, so that a tripartite society is not stable.

5.1 Modeling Cultural Choice

In this subsection, we show how to introduce cultural choice to our model set out in Section 3.

Timing. At the beginning of each period, a player is "exposed" to a randomly selected (non-empty) culture with probability $0 < \theta < 1$. This occurs prior to players being matched in a dyadic interaction. If selected, a player can choose to adopt the culture to which they are exposed or retain their existing culture. Equivalently, we say that the player can migrate to the new cultural community, and we assume that migration is costless.

Cultural Choice. With high probability $(1 - \gamma)$, a $c_{k'}$-member who is exposed to culture c_k, learns: (i) the admissible set of actions for that culture, X_k, (ii) the distribution of players across all cultures at the end of the previous period, denoted by $\nu^t = (n_1^t, n_2^t, \ldots, n_K^t)$, and (iii) a sample of size s drawn (without replacement) from the history $h^t = (h_{j-k}^t)_{j \leq k}$, of the last m actions taken by members of c_j when interacting with c_k-members, for each $j = 1, 2, ...K$. This final piece of information comprises a total of K samples of size s, which together capture how c_k-members have been treated in dyadic interactions by adherents to other cultures, as well as fellow c_k-members. The player is assumed to possess the equivalent information, (i)-(iii), on her present culture. This enables her to calculate and compare expected payoffs between cultures, as follows.

The player's objective is to maximize her expected current period payoff, given that she is unaware who her partner will be in a forthcoming dyadic interaction. She forms expectations based on her information on the history of play, h^t, and the distribution of players across cultures, ν^t. As in a dyadic interaction, the c_k-member uses her sample from h_{j-k}^t to formulate the optimal pure strategy in a j-k interaction, denoted by $\widehat{x}_{k,j}^t(i) \in argmax_{x \in X_k}\{\widehat{p}_{j,k}^t(x) + \delta_{i,x}\}$. Now the player calculates the optimal strategy for each possible culture her partner could adhere to, i.e for all $j \in J = \{1, 2, \ldots K\}$. Since any two players are paired with equal probability, the likelihood of being paired with an adherent to any given culture can be inferred from ν^t. Given the current history of play h^t and distribution of players across cultures ν^t, the expected payoff to a c_k-member before she learns her partner in the dyadic interaction is:

$$\widehat{\pi}^t_{i,k}(h^t, \nu^t) = \sum_{j \in J} \frac{n_j - \xi_j}{N} \{\widehat{p}^t_{j,k}(\widehat{x}^t_{j,k}(i)) + \delta_{i,\widehat{x}^t_{j,k}(i)}\} \tag{9}$$

where $\xi_j = 1$ if $j = k'$ and zero otherwise. If the sum in (9) is strictly greater than the expected payoff from membership in the player's existing cultural community $c_{k'}$, i.e. $\widehat{\pi}^t_{i,k} > \widehat{\pi}^t_{i,k'}$, then the player adopts culture c_k. In the case of a tie, the player chooses one of these two cultures, with uniform probability. Otherwise, she remains an adherent to $c_{k'}$.[18] Recall that all this occurs with probability $(1 - \gamma)$, if a player is selected to review her culture. With low probability γ, the player engages in random cultural experimentation, i.e. she randomly chooses a (non-empty) culture in C.

We assume that once $n^t_k = 0$, $n^{t+b}_k = 0$ for all $b \geq 1$. In other words, once a cultural community becomes empty, it is no longer in the cultural choice set. We have suggested that cultural restrictions arise from the internalization of culturally defined standards of acceptable behavior during socialization. Once a culture has no more adherents, there is no opportunity for social transmission of standards of acceptable behavior. So we assume the culture dies out.[19]

Dynamical Process. With migration between cultural communities, the distribution of players across cultures, ν^t, is now relevant to individual choice. The state in period t, z^t= (h^t, ν^t), specifies the history of play in period t, and the distribution of players across cultures at the end of period $t - 1$. The associated state space is $Z = \mathscr{N} \times \prod_{j \leq k}(X_j \times X_k)^m$, where $\mathscr{N} = \{(n_1, \ldots n_k, \ldots n_K) | \ 0 \leq n_k \leq n, \forall k \in J, and \sum_{k \in J} n_k = n\}$ and $(X)^m$ denotes the m-fold product of X. Z is finite for finite K and m. Again, we assume that the process begins in some initial state z^0 in which there are at least m plays in each j-k interaction, where the sequence of plays is otherwise arbitrary. There are well-defined, time-homogeneous transition probabilities between all pairs of states z, z', denoted by $P_{z,z'}$. Therefore, the adaptive process we have defined is a finite Markov chain, with a $|Z| \times |Z|$ transition probability matrix $P^{m,s,\theta,\varepsilon,\gamma}$. The unperturbed process P^0, now involves $\varepsilon = 0$ and $\gamma = 0$.

5.2 Cultural Configurations in a Tripartite Society

In this section, we analyze the cultural configurations which emerge in the long run, when individuals make a reasoned choice of culture. Our analysis focusses on the

[18]So the choice of culture and its attending commitments are based *partly* on material interests, when interactive payoffs are interpreted as being materially based. There are numerous instances one can cite in which moral sentiments are shaped by material interests. For example, the Puritans who settled Providence Island off the coast of Nicaragua were lured by the sizable profits from trade in plantation crops to abandon their original ideals and become slave owners (Bowles 2004, p.4). Both the temporally dislocated Spartan and Venetian aristocracies took to polyandry in response to the taxation of separate households, which were legally deemed to be created upon marriage. So brothers saved on taxes by sharing a wife (Jones 2006).

[19]Of course, there are cultural storage devices that are external to the physical body, such as written texts, works of art and monuments. Our motivation of cultural restrictions is based on the transmission of standards of behavior from person to person.

following salient case, which we refer to as the *tripartite society*:

TRIPARTITE SOCIETY. *Each dyadic interaction takes the form of a 2×2 coordination game, with pure strategies $X = \{x_1, x_2\}$. There are three cultures to which an individual may adhere: c_1 which permits only action x_1, c_2 which permits only action x_2, and a permissive culture $c_{\mathcal{P}}$ which allows both actions.*

Define $\tilde{Z} = \{z \mid n_k > 0 \text{ for } k = 1, 2, \mathcal{P}\}$, as the set of states in which all three cultures are populated. Let $\mu^t(z|z^0)$ be the relative frequency with which state z is visited by the process P^0 during the first $t > 0$ periods. As t goes to infinity, $\mu^t(z|z^0)$ converges almost surely to the probability distribution $\mu^\infty(z|z^0)$, called the *asymptotic frequency distribution* conditional on z^0. The process selects those states on which $\mu^\infty(z|z^0)$ puts positive probability. We can now state the following result:

Theorem 3 *Consider a tripartite society. If $s/m \leq 1/2$, then for any initial state z^0, the sequence $\mu^t(z|z^0)$ converges almost surely to zero, for all $z \in \tilde{Z}$. In other words, at least one culture dies out in the long run, so that a tripartite society is not stable.*

Proof. Assume $s/m \leq 1/2$. By assumption, once $n_k^t = 0$, $n_k^{t+b} = 0$ for all $b \geq 1$. So no state in $\tilde{Z}$ is accessible from any state in $Z \backslash \tilde{Z}$. Therefore, for P^0 to be in a state $z \in \tilde{Z}$ in period t, the process must have been in a state in $\tilde{Z}$ in every period up to t. To establish the Theorem, it suffices to show that there exists a probability $q > 0$ and a positive integer Q, such that from any state $z \in \tilde{Z}$, P^0 transits with probability at least q to a state in $Z \backslash \tilde{Z}$ in at most Q periods. In this case, the probability of not reaching a state in $Z \backslash \tilde{Z}$ in λQ periods is at most $(1 - q)^\lambda$, which goes to zero as $\lambda \to \infty$. We will now show that such a pair (q, Q) exists.

Assume P^0 is in a state $z \in \tilde{Z}$ in period $t + 1$. Since in such a state, $n_k^{t+1} > 0$ for all $k \in \{1, 2, \mathcal{P}\}$, there is a positive probability of any possible *j*-*k* interaction occurring. Suppose that no player is selected to review her culture in $Q' = t + 3m + 12s$ consecutive periods, beginning in period $t + 1$. Recall that the probability with which a player is selected to review her culture in a given period is $\theta < 1$, so this event occurs with positive probability. There is also a positive probability that in each period from $t + 1$ to $t + m$, both players matched in a dyadic interaction are c_1-members. Since all c_1-members are restricted to taking actions in $X_1 = \{x_1\}$, the 1-1 history transits with probability one to the x_1 convention $((x_1, x_1), \ldots (x_1, x_1))$ by the end of period $t + m$. There is a positive probability then that in each period from $t + m + 1$ to $t + 2m$, both players matched in a dyadic interaction are c_2-members. Since $X_2 = \{x_2\}$, the 2-2 history transits with probability one to the x_2 convention by the end of period $t + 2m$. There is a positive probability then that a c_1-member and a c_2-member are matched in a dyadic interaction, in each period from $t + 2m + 1$ to $t + 3m$. Since $X_1 \cap X_2 = \emptyset$, the 1-2 history transits with probability one, by the end of period $t + 3m$, to the state of miscoordination $((x_1, x_2), (x_1, x_2), \ldots, (x_1, x_2))$.

Since $X_{\mathcal{P}} = \{x_1, x_2\} = X$, $\tilde{X}_k \cap X_{\mathcal{P}} \neq \emptyset$, for $k = 1, 2$. Therefore, neither cultures c_1 and $c_{\mathcal{P}}$, or c_2 and $c_{\mathcal{P}}$ are weakly opposed. Now consider the event in which a c_1-member and a $c_{\mathcal{P}}$-member are matched in each period from $t + 3m + 1$ to $t + 4m +$

$2s$. Since by hypothesis there is no migration of players in this time interval, we can directly apply the argument used in the proof of Lemma 2, case 1 (see the Appendix), to show that there is a positive probability that the 1-$\mathcal{P}$ history is an x_1 convention, by the end of period $t + 4m + 2s$ (i.e. in at most $2s + m$ periods). The same argument can be applied to show that there is a positive probability that the 2-$\mathcal{P}$ history is an x_2 convention, by the end of period $t + 5m + 4s$. Since $X_{\mathcal{P}} \subseteq X_{\mathcal{P}}$, cultures $c_{\mathcal{P}}$ and $c_{\mathcal{P}}$ are not weakly opposed. By Corollary 1, play in $\mathcal{P}$-$\mathcal{P}$ interactions cannot end in recurrent miscoordination. So there is a positive probability that from any $\mathcal{P}$-$\mathcal{P}$ history in period $t + 5m + 4s + 1$, the $\mathcal{P}$-$\mathcal{P}$ history is an x_ℓ convention, $\ell \in \{1, 2\}$, by the end of period $t + 6m + 6s$. The joint probability of all this occurring is positive.

We have already established that condition ND guarantees that no matter which individuals are selected to play, x_ℓ is a best reply to all samples from an x_ℓ convention. Therefore, whenever migration between cultural communities occurs, the conventions in 1-1, 2-2, 1-$\mathcal{P}$, 2-$\mathcal{P}$ and $\mathcal{P}$-$\mathcal{P}$ interactions remain in place. Since $X_1 \cap X_2 = \emptyset$, the 1-2 history remains in a state of miscoordination, no matter who is selected to play. The expected interactive payoff for any sample from such a 1-2 history is zero. Therefore, there are in effect two different types of overall histories; one in which an x_1 convention is in place in $\mathcal{P}$-$\mathcal{P}$ interactions and one in which an x_2 convention is in place. Denote the former by h_1^* and the latter by h_2^*.

Suppose that migration between cultural communities occurs in some period $t' > t + 6m + 6s$. A player selected to review her culture is randomly "exposed" to another culture, and can choose whether to adopt that culture. For convenience denote $n_k^{t'}$ by n_k, for the moment. Recall that in period t', $n_k > 0$ for all $k = 1, 2, \mathcal{P}$. Also, recall that each pair of players is drawn with equal probability, and $n = n_1 + n_2 + n_{\mathcal{P}}$. Consider a player i who is a c_1-member at the start of period t'. If i remains a c_1-member in period t' and she is matched in a dyadic interaction, then i is paired with a c_1-member or a $c_{\mathcal{P}}$-member with probability $\frac{n_1 - 1 + n_{\mathcal{P}}}{n-1}$. Recall that an x_1 convention $((x_1, x_1), \ldots (x_1, x_1))$ is in place in 1-1 and 1-$\mathcal{P}$ interactions under both h_1^* and h_2^*. So all c_1-members expect to receive an interactive payoff of one in 1-1 and 1-$\mathcal{P}$ interactions. Since 1-2 interactions are perpetually in a state of miscoordination, a c_1-member always expects to receive a zero interactive payoff in 1-2 interactions. Therefore, the expected payoff to i if she remains a c_1-member in period t' (and is thus restricted to playing x_1), is $\delta_{i,x_1} + \frac{n_1 - 1 + n_{\mathcal{P}}}{n-1}$, under both h_1^* and h_2^*. Similarly, if player i is a c_2-member at the start of period t', her expected payoff from retaining c_2 is $\delta_{i,x_2} + \frac{n_2 - 1 + n_{\mathcal{P}}}{n-1}$, under both h_1^* and h_2^*. Player i's expected payoff from membership in $c_{\mathcal{P}}$ is $(1 + \frac{n_1 - (1-\xi) + \beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2 - \xi + (1-\beta) n_{\mathcal{P}}}{n-1}\delta_{i,x_2})$, where $\beta = 1$ (resp. $\beta = 0$) indicates here that the $\mathcal{P}$-$\mathcal{P}$ convention is x_1 (resp. x_2), and $\xi = 1$ (resp. $\xi = 0$) indicates that player i adheres to c_2 (resp. c_1) at the beginning of period t'. As such, if i is a c_1-member at the beginning of period t', membership in $c_{\mathcal{P}}$ yields a strictly greater expected payoff than c_1, if and only if:

$$\delta_{i,x_1} + \frac{n_1 - 1 + n_{\mathcal{P}}}{n-1} < 1 + \frac{n_1 - 1 + \beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2 + (1-\beta) n_{\mathcal{P}}}{n-1}\delta_{i,x_2} \tag{10}$$

If i is a c_2-member at the beginning of period t', membership in $c_{\mathcal{P}}$ yields a strictly greater expected payoff than membership in c_2, if and only if:

$$\delta_{i,x,2} + \frac{n_2 - 1 + n_{\mathcal{P}}}{n-1} < 1 + \frac{n_1 + \beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2 - 1 + (1-\beta)n_{\mathcal{P}}}{n-1}\delta_{i,x_2} \tag{11}$$

If $\beta = 1$, then we can rearrange (10) as follows:

$$n_2(\delta_{i,x_1} - \delta_{i,x_2}) < n_2 \tag{12}$$

Under condition ND, $|\delta_{i,x_1} - \delta_{i,x_2}| < 1$ for all $i \in N$. Therefore, membership in $c_{\mathcal{P}}$ yields a strictly greater expected payoff than c_1 for all $i \in N$ (since $n_2 \neq 0$). There is a positive probability that all players who are c_1-members at the start of period t', are exposed to culture $c_{\mathcal{P}}$ in the $n_1^{t'}$ consecutive periods from period $t'+1$ to $t'+n_1^{t'}$, and that each $i \in N_1$ chooses to adopt $c_{\mathcal{P}}$. So from any cultural distribution, $(n_1^{t'}, n_2^{t'}, n_{\mathcal{P}}^{t'}) \gg 0$, there is a positive probability that $n_1 = 0$ in period $t' + n_1^{t'}$. When $\beta = 0$, (11) can be rearranged to show that there is a positive probability that $n_2 = 0$ in period $t' + n_2^{t'}$. The joint probability of all this occurring is positive. Therefore, there exists a probability $q > 0$ and a positive integer Q, such that from any state $z \in \tilde{Z}$, P^0 transits with probability at least q to a state in $Z \backslash \tilde{Z}$ in at most Q periods. This establishes the Theorem. □

We have shown that it is possible for conventions to emerge in intercultural interactions, even where there are more than two cultures to begin with, and when players can migrate between cultural communities. Nevertheless, it is always the case that at most two cultures survive in the long-run. This is a surprising result. Given sufficient heterogeneity in individual preferences, we might expect *a priori* that all three cultures are populated in the long run. On the contrary, we have shown that even with almost unrestricted heterogeneity in individual preferences, a tripartite society is not stable.[20] We remark that any of the remaining cultural configurations can emerge in the long run, including the case in which all players adhere to the permissive culture or players are divided between the two restrictive cultures. We plan to develop the stochastic stability analysis of the coevolution of coordination and cultural choice in future work to select between these cultural configurations. We also plan to extend the analysis to more general cases than the tripartite society, in which there are more than two totally restrictive cultures and there are cultures with intermediate restrictiveness.

[20] Recall that we have only imposed the rather mild condition ND on the distribution of individual preferences.

Appendix

Proof of Lemma 2

To establish the Lemma, it suffices to show that there is a positive probability of transiting from any state to a convention or a state in $\mathcal{M}$ in a finite number of periods. Consider the situation at some arbitrary time $t+1$. There is a positive probability that a particular player $i \in N_1$ is selected to play in s consecutive periods, and that i draws the same sample, which we denote by $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, on each occasion (since $m \geq 2s$). There is also a positive probability that i plays the same best reply, say x_ℓ, in the s consecutive periods from period $t+1$ to $t+s$. So in period $t+s+1$, the c_2-member sampling from the most recent m plays by a c_1-member up to period $t+s$, has a positive probability of drawing the sample $(x_\ell, x_\ell, \ldots x_\ell)$, comprised solely of s plays of x_ℓ. There are then two cases to consider.

Case 1. $x_\ell \in X_1 \cap X_2$. (By definition, if c_1 and c_2 are radically opposed, then $x_\ell \in X_1 \cap X_2$ is impossible.) Condition ND guarantees that x_ℓ is the unique best reply to the sample $(x_\ell, x_\ell, \ldots x_\ell)$ for all c_2-members. There is a positive probability that player $i \in N_1$ is again selected to play in period $t+s+1$, that she draws the same sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$ [since $m \geq 2s$], and that she plays the same best reply x_ℓ to this sample. Therefore, there is a positive probability that the action-tuple played in period $t+s+1$ will be (x_ℓ, x_ℓ). Since at the end of this period, the last s consecutive plays by the c_1-member are of x_ℓ, the c_2-member selected in period $t+s+2$ has a positive probability of again drawing a sample comprised of s plays of x_ℓ, to which the unique best reply is x_ℓ.

Now consider the c_1-member. There is a positive probability that player $i \in N_1$ is again selected to play in period $t+s+2$. Denote the history of the most recent m plays by c_2-members in period $t+s+1$ by $(x^1, x, \ldots)$. Once the c_2-member drawn in period $t+s+1$ plays action x_ℓ, the corresponding period $t+s+2$ history is $(x, \ldots x_\ell)$. The only change in the history is that x^1 is dropped and x_ℓ is added. If $x^1 \neq x^{-s}$, then player i can once again draw the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, and play the same best reply x_ℓ to this sample. If $x^1 = x^{-s}$, then i can draw the sample $(x^{-s+1}, \ldots x^{-1}, x_\ell)$, which contains at least as many plays of x_ℓ as the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$. Therefore, if x_ℓ is a best reply to the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, then it is also a best reply to $(x^{-s+1}, \ldots x^{-1}, x_\ell)$. As such, there is a positive probability that the action-tuple played in period $t+s+2$ is again (x_ℓ, x_ℓ). Iterating this argument, the process P^0_{1-2} transits with positive probability to the convention $h^*_\ell = ((x_\ell, x_\ell), \ldots (x_\ell, x_\ell))$ in at most $m-2$ additional periods. Notice that we have not relied on the cultures not being weakly opposed, at any stage. So this result applies whenever c_1 and c_2 are weakly, but not radically opposed.

Case 2(a). $x_\ell \notin X_1 \cap X_2$ and c_1 and c_2 are not weakly opposed. Then there exists an individual $i' \in N_2$, such that $\tilde{x} \in argmax_{x \in X_2} \delta_{i',x}$ is in $X_1 \cap X_2$. There is a positive probability that $i' \in N_2$ is selected in period $t+s+1$, and draws the sample $(x_\ell, x_\ell, \ldots x_\ell)$. For this sample, $\widehat{p}_{1,2}(x_\ell) = 1$. Since $x_\ell \in X_1$ (because x_ℓ was taken by a c_1-member), and $x_\ell \notin X_2$ (because $x_\ell \in X_1$ and $x_\ell \notin X_1 \cap X_2$), $\widehat{p}_{1,2}(x) = 0$ for all $x \in X_2$. Therefore, the expected payoff from any admissible action $x \in X_2$ for i' is $\widehat{p}_{1,2}(x) + \delta_{i',x} = \delta_{i',x}$. As such, the best reply to this sample for i' involves simply maximizing her idiosyncratic payoff by choosing an action in $argmax_{x \in X_2} \delta_{i',x}$. By assumption there exists such an action, say $\tilde{x}$, in $X_1 \cap X_2$. So there is a positive probability that i' plays action $\tilde{x}$ in period $t+s+1$. There is also a positive probability that i' is selected to play in the next s consecutive periods, that she draws the same sample $(x_\ell, x_\ell, \ldots x_\ell)$ in each of those periods (since $m \geq 2s$), and that she plays the same best reply $\tilde{x}$ on each occasion. Then in period $t+2s+1$, there is a positive probability that the c_1-member selected to play draws the sample $(\tilde{x}, \tilde{x}, \ldots \tilde{x})$. Since $\tilde{x} \in X_1 \cap X_2$, we are now in case 1, and there is a positive probability of transiting to the $\tilde{x}$ convention in at most another m periods.

Case 2(b). $x_\ell \notin X_1 \cap X_2$ and c_1 and c_2 are weakly opposed. Again, there is a positive probability that the c_2-member i' selected in period $t+s+1$ plays the best reply $\tilde{x} \in argmax_{x \in X_2} \delta_{i',x}$ to the sample $(x_\ell, x_\ell, \ldots x_\ell,)$. Only now $\tilde{x} \notin X_1 \cap X_2$. Consider the c_1-member. There is a positive probability that player $i \in N_1$ is again selected in period $t+s+1$, that i draws the same sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$ [since $m \geq 2s$], and that she again plays the best reply x_ℓ. Therefore, there is a positive probability that the action-tuple played in period $t+s+1$ is $(x_\ell, \tilde{x})$.

In period $t+s+2$, there is a positive probability that player $i' \in N_2$ is again selected to play. Since in this period, the last s consecutive plays by the c_1-member are of x_ℓ, there is a positive probability that i' again draws a sample comprised of s plays of x_ℓ, and that she again plays $\tilde{x}$ in response. There is also a positive probability that player $i \in N_1$ is again selected to play in period $t+s+2$. Denote the history of the most recent m plays by c_2-members in period $t+s+1$ by $(x^1, x, \ldots)$. Once the c_2-member drawn in period $t+s+1$ plays action $\tilde{x}$, the corresponding period $t+s+2$ history is $(x, \ldots \tilde{x})$. The only change in the history is that x^1 is dropped and $\tilde{x}$ is added. If $x^1 \neq x^{-s}$, then player i can once again draw the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, and play the same best reply x_ℓ to this sample. If $x^1 = x^{-s}$, then i can draw the sample $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$. Since $\tilde{x} \notin X_1$, this implies that the sample $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$ does not contain a greater frequency of any action $x \in X_1$ than does $(x^{-s}, x^{-s+1}, \ldots x^{-1})$. Since $x_\ell \notin X_2$ by hypothesis, the expected payoff from action x_ℓ is always δ_{i,x_ℓ}. Together with the preceding, this implies that if x_ℓ is a best reply for i to the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, then it is also a best reply to $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$. As such, there is a positive probability that the action-tuple played in period $t+s+2$ is again $(x_\ell, \tilde{x})$. Iterating this argument, the process P^0_{1-2} transits with positive probability to the state $((x_\ell, \tilde{x}), \ldots (x_\ell, \tilde{x}))$ in at most another $s-2$ periods. Since $x_\ell \notin X_2$ and $\tilde{x} \notin X_1$, this is a state of miscoordination. All best replies to a state of miscoordination are in $\tilde{X}_1 \times \tilde{X}_2$. Therefore, P^0_{1-2} transits from this point to a state in $\mathcal{M}$ with probability one in at most m additional periods. $\square$

Proof of Corollary 1

Assume $s/m \leq 1/2$. Without loss of generality, fix $k=1$ and consider an x_1 convention. We have already established in the proof of Lemma 1 part (i), that x_1 is the unique best reply for all possible samples from an x_1 convention, *no matter which players are selected to play*. So in a 1-1 interaction, each convention is an absorbing state and hence a distinct recurrence class of P^0_{1-1}. To establish the corollary then, it will (as before) suffice to show that P^0_{1-1} transits with positive probability from any state to a convention in a finite number of periods. Consider the event in which player $i \in N_1$ is selected to fill the first player position in $s+m$ consecutive periods and player $i' \in N_1$ is selected to fill the second player position in the same $s+m$ periods. This event occurs with positive probability, since in a 1-1 interaction each player in N_1 is drawn with positive probability and then randomly allocated to the first or second player position. Therefore, the pairing in this manner of i and i' in $s+m$ consecutive periods, is analogous to the case in which i and i' are drawn from two disjoint populations and belong to separate cultural communities c_1 and $c_{1'}$, respectively, with admissible action sets X_1 and $X_{1'} = X_1$. There is a positive probability that i draws the same sample in the first s periods, and plays the same best reply, say $x_\ell \in X_1$ on each occasion. Since $X_1 = X_{1'}$, x_ℓ is certainly in $X_1 \cap X_{1'}$. Therefore, we can directly apply the argument used in the proof of Lemma 2 (case 1) to show that from this point, the process converges with positive probability to the x_ℓ convention in at most m additional periods. □

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